Vortex structure in long Josephson junction with two inhomogeneities

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A report of numerical experiment results on long Josephson junction with one and two rectangular inhomogeneities in the barrier layer is presented. In case of one inhomogeneity we demonstrate the existence of the asymmetric fluxon states. The disappearance of mixed fluxon-antifluxon states when the position of inhomogeneity shifted to the end of the junction is shown. In case with two inhomogeneities the change of the amplitude of Josephson current through the inhomogeneity at the end of junction makes strong effect on the stability of the fluxon states and smoothes the maximums on the dependence "critical current - magnetic field". Pacs: 05.45.+b74.50.+r 74.40.+k. Keywords: long Josephson junction, inhomogeneity, bifurcation, critical curve

In order to investigate a stability of fluxon states in inhomogeneous in-line Josephson junction (JJ), we solve the following non-linear eigenvalue problem

$$-\varphi_{xx} + j_C(x)\sin\varphi = 0, \tag{1a}$$

$$\varphi_x(0) = h_e - \varkappa_l\, L\gamma,\ \varphi_x(L) = h_e + \varkappa_r\, L\gamma, \tag{1b}$$

$$-\psi_{xx} + \psi\, j_C(x)\,\cos\varphi = \lambda\,\psi, \tag{1c}$$

$$\psi_x(0) = 0,\ \psi_x(l) = 0,\ \int\limits_0^L \psi^2(x)\,dx - 1 = 0, \tag{1d}$$

with respect to the triplet $\{\varphi(x), \psi(x), h_e\}$. Here $\varphi(x)$ represents the static magnetic distribution in JJ, L is junction's length, h_e — external magnetic field, $\varkappa_l + \varkappa_r = 1$.

The inhomogeneity in the form of the narrow rectangular well is characterized by its width $\Delta < L$, localization $\zeta \in [\Delta/2, L-\Delta/2]$ and portion of Josephson current κ through it. An existence of the inhomogeneity leads to the local change of the Josephson current, which is equal to $j_C(x) = 1+\kappa$ inside of the inhomogeneity and $j_C(x) = 1$ outside of it. At $\kappa > 0$ we have shunt, at $\kappa \in [-1, 0)$ — microresistor. The value $\kappa = 0$ corresponds to the homogeneous junction. A minimal eigenvalue of the Sturm-Liouville problem allow to make a conclusion about the stability of the $\varphi(x)$ (see details in[1] –[4]).

Results of numerical solution of non-linear eigenvalue problem (1) in the in-line geometry is presented in Fig. **??**, where we demonstrate an influence of the inhomogeneity position on the bifurcation curves "critical current – external magnetic field" for $L = 7$ and $\Delta = 0.7$. As we can see, the inhomogeneity in the center of JJ leads to the appearance of stable mixed states like $\Phi^n\Phi^m$ ($n, m = \pm1, \pm2, \ldots$, $n \neq m$ and $nm < 0$) and to the non-monotonic decrease of

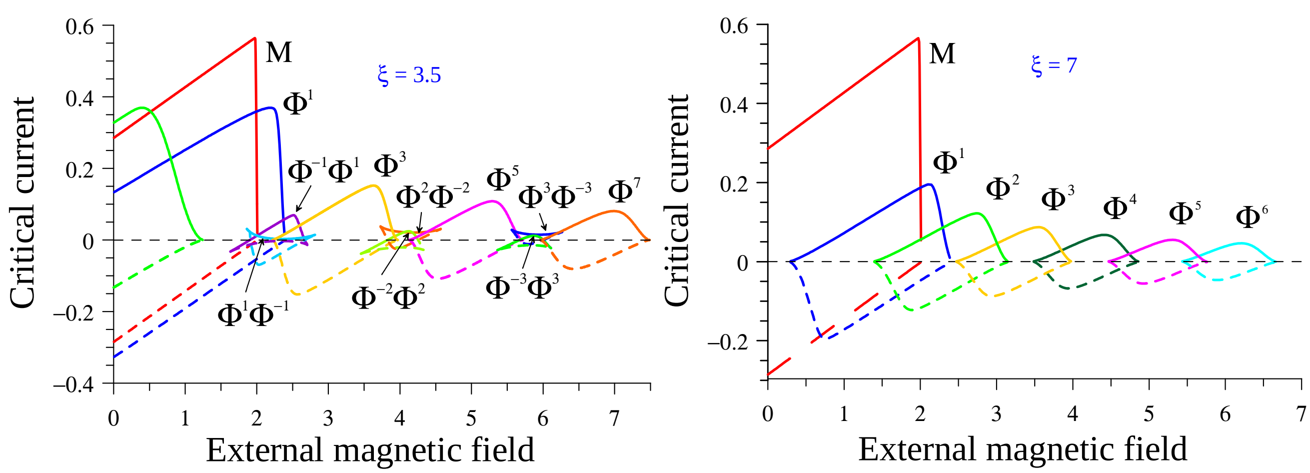

FIG. 1: Critical curve of the junction with the inhomogeneity at the center of junction (left) and at the end of the junction(right). The length of the junction is $L = 7$ and width of inhomogeneity is $\Delta = 0.7$

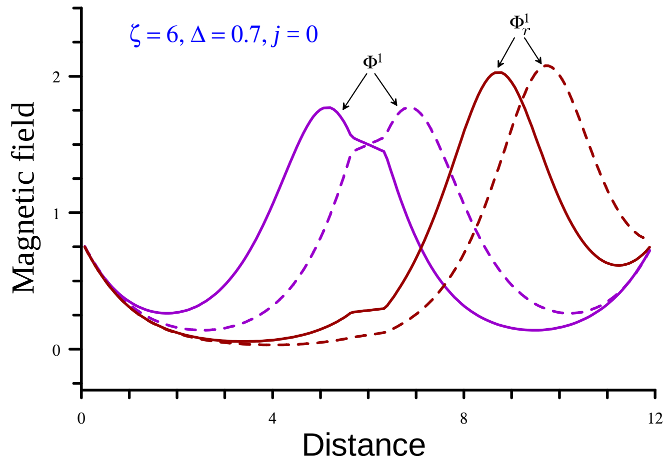

FIG. 2: The distribution of the magnetic field along the junction for Φ^1 and Φ^1_r at $h_e = 0.8$ and $\gamma = 0$

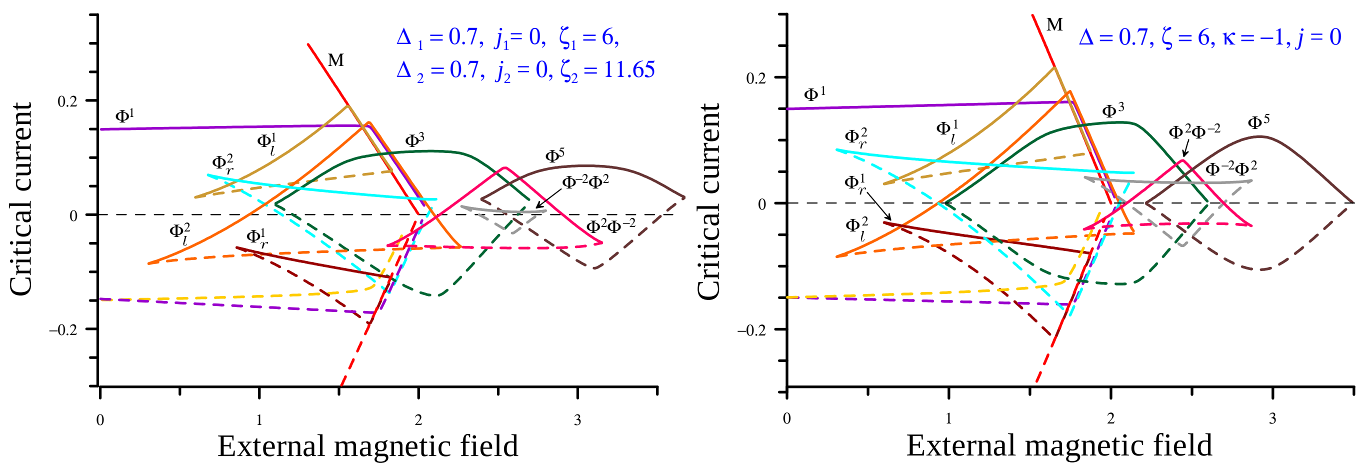

FIG. 3: Critical curves for the junction with one inhomogeneity at the center with $j_{C1} = 0$ and another one at the end of the junction with $j_{C2} = 1$ (a) and $j_{C2} = 0$ (b) . The length of the junction is $L = 12$ and width of inhomogeneities are $\Delta = 0.7$

maximums of critical curves for pure fluxon states with magnetic field. Here $\Phi^{\pm 1}$ denotes single fluxon (antifluxon) state in the junction. In Fig. **??** (right) we demonstrate a disappearance of mixed fluxon-antifluxon states when the position of inhomogeneity is shifted to the end of the junction. The same result we observed in the overlap geometry.

In long JJ with the inhomogeneity in the center of junction in addition to the central fluxon states appear the asymmetric fluxon states. In the overlap geometry for the junction with $L = 12$, width of inhomogeneity $\Delta = 0.7$ and the amplitude of Josephson current through inhomogeneity $j_C = 0$ we have observed the "left" and the "right" fluxon states Φ^1_l, Φ^1_r, $(\Phi^2\Phi^{-2})_l$ and $(\Phi^{-2}\Phi^2)_r$. The distribution of the magnetic field along the junction for Φ^1 and Φ^1_r is shown in Fig. 2.

We compare the bifurcation curves for the junction with one inhomogeneity at the center with a case of JJ with two resistive inhomogeneities. The position of the first one was fixed in the center and second one at the right end of the junction. The result of simulation for $\Delta_1 = \Delta_2 = 0.7$ and $j_{C1} = j_{C2} = 0$ is presented in Fig. 3. We study the influence of the value j_{C2} on the bifurcation curves of the junction and get the next main results. The decrease of the current through the inhomogeneity on the right side of the junction makes a weak influence on the curves Φ^1_l and Φ^2_l but essentially changes the curves Φ^1_r and Φ^2_r. There is strong effect on the bifurcation curves of mixed states: stability region for $\Phi^2\Phi^{-2}$ is increased, and for $\Phi^{-2}\Phi^2$ is decreased. We observe that this decrease in j_{C2} smoothes the maximums on the dependence "critical current - magnetic field".

The bifurcation curves for mixed states demonstrate an interesting peculiarity. In some intervals of magnetic field these states are stable only if the current through junction is not equal to zero. We call such intervals as CC-regions (created by current). When the inhomogeneity is shifted from the center of the junction, the pure fluxon states have the CC-regions as well. In our case with two inhomogeneities we have observed the appearance of CC-regions for pure

fluxon states by the decrease of the current through the second inhomogeneity. We found also that Φ_l^1 and Φ_r^1 are not stable without current at all.

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